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\mathbb{Z}^2 is a free \mathbb{Z} -module of rank 2. Let \mathcal{L} be a lattice in \mathbb{Z}^2 . The quotient $\mathbb{Z}^2 / \mathcal{L}$ is a finite abelian group. The order of $\mathbb{Z}^2 / \mathcal{L}$ is the area of the fundamental parallelogram of \mathcal{L} . If \mathcal{L} is generated by $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then the order is $|\det \begin{pmatrix} a & b \\ c & d \end{pmatrix}|$. For example, if $\mathcal{L} = 2\mathbb{Z} \times 2\mathbb{Z}$, then the order is $2 \times 2 = 4$.

The index of a sublattice \mathcal{L}' of \mathcal{L} is the order of the quotient $\mathcal{L} / \mathcal{L}'$. If \mathcal{L}' is generated by $\begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}$, then the index is $|\det \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}| / |\det \begin{pmatrix} a & b \\ c & d \end{pmatrix}|$.

1. Let \mathcal{L} be the lattice in \mathbb{Z}^2 generated by $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Let \mathcal{L}' be the sublattice of \mathcal{L} generated by $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. The index of \mathcal{L}' in \mathcal{L} is $|\det \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}| / |\det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}| = 4$.

2. ...

3. ...

4. Let \mathcal{L} be the lattice in \mathbb{Z}^2 generated by $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Let \mathcal{L}' be the sublattice of \mathcal{L} generated by $\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$. The index of \mathcal{L}' in \mathcal{L} is $|\det \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}| / |\det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}| = 2$.

5. Let \mathcal{L} be the lattice in \mathbb{Z}^2 generated by $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Let \mathcal{L}' be the sublattice of \mathcal{L} generated by $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$. The index of \mathcal{L}' in \mathcal{L} is $|\det \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}| / |\det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}| = 1$.

Let \mathcal{L} be the lattice in \mathbb{Z}^2 generated by $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Let \mathcal{L}' be the sublattice of \mathcal{L} generated by $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$. The index of \mathcal{L}' in \mathcal{L} is $|\det \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}| / |\det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}| = 1$.

Let \mathcal{L} be the lattice in \mathbb{Z}^2 generated by $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Let \mathcal{L}' be the sublattice of \mathcal{L} generated by $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$. The index of \mathcal{L}' in \mathcal{L} is $|\det \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}| / |\det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}| = 1$.